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GORENSTEIN PROPERTIES OF SPLIT-BY-NILPOTENT EXTENSION ALGEBRAS

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ABSTRACT. Let A be a finite-dimensional k-algebra over an algebraically closed field k. In this note, we study the Gorenstein homological properties of a split-by-nilpotent extension algebra. Let R be a split-by-nilpotent extension of A. We provide sufficient conditions to ensure when a Gorenstein-projective module over A induces a similar structure over R. We also study when a Gorenstein-projective R-module induces a Gorenstein-projective A-module. Moreover, we study the relationship between the Gorensteinness of A and R.

INTRODUCTION

Gorenstein homological algebra has been thoroughly studied in the last decades. These algebras played an important role in the representation theory of finite-dimensional algebras. In particular, for the setting of cluster theory, we highlight the work of B. Keller and I. Reiten [12], where they proved that all cluster algebras are Gorenstein. It is well known also that all gentle algebras are Gorenstein, see [10]. In [15], C. M. Ringel studied indecomposable Gorenstein-projective modules over Nakayama algebras and characterized Nakayama algebras that are Gorenstein. In a similar direction, in [7] the authors classified Gorenstein-projective modules over monomial algebras.

Finitely generated Gorenstein-projective modules over a noetherian ring were introduced by M. Auslander and M. Bridger [3]. More generally, the notion of Gorenstein-projective modules over an arbitrary ring was defined by E. Enochs and O. Jenda in [8]. Consequently, if we construct the dual of those modules we obtain Gorenstein-injective modules, which where also developed in [8].

It is not clear that classical constructions over algebras preserve Gorenstein properties. Under certain hypotheses, it is possible to define new Gorenstein algebras from the initial ones, see for example [6], [13], or [18].

In this work, we focus our attention on Gorenstein-projective modules over split-by-nilpotent extension algebras.

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Let A be a finite-dimensional k-algebra over an algebraically closed field k and R a split-extension of A by the nilpotent ideal E. A recurrent problem of split-by-nilpotent extension algebras is to predict the properties that R inherits from the preceding algebra. The principal tool used in this work to study the connection between the categories $\operatorname{mod} A$ and $\operatorname{mod} R$ is the theory of change of rings functors. The main result of our work is the following.

Theorem A (Corollary 2.2)**.** *Let R be a split-by-nilpotent extension of A by the nilpotent ideal E.*

(i) *Assume that* $\operatorname{pd}_A R < \infty$, $\operatorname{id}_A R < \infty$, $\operatorname{pd}_A DR < \infty$ *and* $\operatorname{id}_A DR < \infty$. *If N is a Gorenstein-projective A-module then $R \otimes_A N$ is a Gorenstein-projective R-module.*

(ii) *Assume that* $\operatorname{pd}_R A < \infty$, $\operatorname{id}_R A < \infty$, $\operatorname{pd}_R DA < \infty$ *and* $\operatorname{id}_R DA < \infty$. *If L is a Gorenstein-projective R-module then $A \otimes_R L$ is a Gorenstein-projective R-module.*

We obtain a similar result to the one stated in Theorem A for Gorenstein-injective modules.

Finally, it is of particular interest in the representation theory of artin algebras the study of Gorenstein-projective modules over Gorenstein algebras. By assuming some hypotheses, as we show below in Theorem B, it is possible to determine that the extension always inherits the quality of A of being Gorenstein.

Theorem B (Theorem 2.3)**.** *Let R be a split-by-nilpotent extension of A by the nilpotent ideal E. Assume that* $\operatorname{pd}_R E < \infty$, $\operatorname{pd}_A E < \infty$, $\operatorname{id}_R DE < \infty$ *and* $\operatorname{id}_A DE < \infty$. *Then A is Gorenstein if and only if R is Gorenstein.*

The note is organized as follows. After a brief section of preliminaries, we devote Section 2 to the proofs of Theorems A and B.

1. Preliminaries

Throughout this note, all algebras are associative basic connected finite-dimensional over an algebraically closed field k. For an algebra A, we denote by $\operatorname{mod} A$ the category of finitely generated left A-modules.

We denote by D the usual standard duality $\operatorname{Hom}_k(-, k) : \operatorname{mod} A \to \operatorname{mod} A^{op}$, see [2, I, 2.9].

1.1. Gorenstein-projective modules.

A complex $C^\bullet$ of A-modules is *acyclic* provided that it is exact as a sequence or, equivalently, $H^n(C^\bullet) = 0$ for all n. It follows from [4, p. 400] that a complex $P^\bullet$ of projective A-modules is *totally acyclic* provided it is acyclic and the Hom complex $\operatorname{Hom}(P^\bullet, A)$ is also acyclic.

Following E. Enochs and O. Jenda [9], we have the following definition.

Definition 1.1. An A-module M is *Gorenstein-projective* provided that there is a totally acyclic complex $P^\bullet$ of projective modules such that its 0-th cocycle $Z^0(P^\bullet)$ is isomorphic to M.

Let us denote by A-Gproj the full subcategory of $\operatorname{mod} A$ consisting of Gorenstein-projective modules. In Definition 1.1, the complex $P^\bullet$ is said to be a *complete resolution* of M. Note that any projective module P is Gorenstein-projective. Therefore, A-proj $\subset$ A-Gproj. If we consider the dual of Gorenstein-projective modules we obtain the notion of Gorenstein-injective modules, see [9].

Following [16], a module M is said to be *semi-Gorenstein-projective* provided that $\operatorname{Ext}^i_A(M, A) = 0$ for all $i \geq 1$. All Gorenstein-projective modules are semi-Gorenstein-projective modules. If we denote by ${}^\perp A$ the class of all semi-Gorenstein-projective modules, then A-Gproj $\subset {}^\perp A$. It is well known that a module M is a Gorenstein-projective module if and only if both M and $\operatorname{Tr} M$ are semi-Gorenstein-projective modules, see for example [16]. In [14], R. Marczinzik presented examples of non-commutative algebras with semi-Gorenstein-projective modules which are not Gorenstein-projective modules. The following result shall be helpful for our purpose.

Lemma 1.2. *Let $M \in {}^\perp A$ and $L, L' \in \operatorname{mod} A$.*

(i) *If $\operatorname{pd}_A L < \infty$ then $\operatorname{Ext}^i_A(M, L) = 0$ for all $i \geq 1$.*
(ii) *If $\operatorname{id}_A L' < \infty$ then $\operatorname{Tor}^i_A(L, M) = 0$ for all $i \geq 1$.*

Recall that an artin algebra A is *Gorenstein* provided that the regular module A has finite injective dimension on both sides, see [11].

It is well known that if A is an artin algebra then it is Gorenstein if and only if $\operatorname{id}_A A < \infty$ and $\operatorname{pd}_A DA < \infty$.

Proposition 1.3 ([9, Proposition 9.1.7])**.** *Let A be a Gorenstein algebra and $M \in \operatorname{mod} A$. Then M has finite projective dimension if and only if M has finite injective dimension.*

1.2. **Split extensions of algebras by a nilpotent ideal.** Given two algebras A and R and a surjective algebra homomorphism $\pi : R \to A$, the morphism π is said to be *split* if there exists an algebra homomorphism $\sigma : A \to R$ such that $\pi\sigma = \operatorname{Id}_A$. In this situation, the kernel E of π is a two-sided ideal in R. Therefore we have a short exact sequence of abelian groups

$$0 \to E \xrightarrow{i} R \underset{\sigma}{\overset{\pi}{\rightleftarrows}} A \to 0, \tag{1.1}$$

where i denotes the inclusion morphism. Since $\pi\sigma = \operatorname{Id}_A$, we have that σ is injective and thus A is isomorphic to a subalgebra of R. In particular, E inherits an A-A-bimodule structure by restriction of scalars. Therefore, (1.1) is a split sequence of A-modules, which allows us to find an A-module isomorphism, $R \cong A \oplus E$.

Definition 1.4. Let A, R be algebras. We say that R is a *split extension* of A by a nilpotent ideal E, or a *split-by-nilpotent extension* for short, if there exists a split surjective algebra homomorphism $\pi : R \to A$ whose kernel is E.

Let R be a split-by-nilpotent extension of A by the nilpotent ideal E. Then we have two categories $\operatorname{mod} A$ and $\operatorname{mod} R$. A problem of interest in the theory of split extensions is to compare these two categories. With this purpose we define both changes of ring functors as

$$R \otimes_A - : \operatorname{mod} A \to \operatorname{mod} R$$

and

$$A \otimes_R - : \operatorname{mod} R \to \operatorname{mod} A.$$

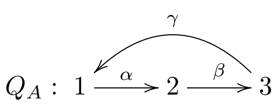
These functors satisfy the following adjunction relation: $A \otimes_R R \otimes_A - \cong \mathrm{Id}_{\operatorname{mod} A}$. The reverse composition of these functors, in general, is not equal to the identity in $\operatorname{mod} R$.

In [1, Lemma 1.2], I. Assem and N. Marmaridis proved that there is a bijective correspondence between the isoclasses of indecomposable projective A-modules and the isoclasses of indecomposable projective R-modules.

Proposition 1.5 ([17, Proposition 4.4])**.** *Let R be a split-by-nilpotent extension of A by the nilpotent ideal E. The following conditions hold:*

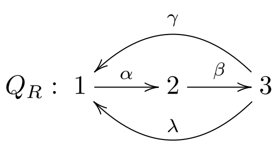
- (i) *If $L \in \operatorname{mod} A$, then*
  - (a) $\operatorname{pd}_A L \leq \operatorname{pd}_R L + \operatorname{pd}_A E$*;*
  - (b) $\operatorname{id}_A L \leq \operatorname{id}_R L + \operatorname{id}_A DR$*.*
- (ii) *If $N \in \operatorname{mod} R$, then*
  - (a) $\operatorname{pd}_R N \leq \operatorname{pd}_A N + \operatorname{pd}_R A$*;*
  - (b) $\operatorname{id}_R N \leq \operatorname{id}_A N + \operatorname{id}_R DA$*.*

Remark 1.6. In general, for $L \in \operatorname{mod} R$ it is not true that if $\operatorname{pd}_R L < \infty$ then $\operatorname{pd}_A L < \infty$. For example, consider the following algebras:



$$I_A = \langle \alpha\gamma\beta\alpha \rangle, \quad A = kQ_A/I_A$$

and



$$I_R = \langle \alpha\gamma\beta\alpha, \alpha\lambda \rangle, \quad R = kQ_R/I_R.$$

Observe that R is a split extension of A by the ideal generated by λ. Consider L the projective R-module corresponding to the vertex 3. Then, as an A-module, L is isomorphic to $P_3^A \oplus S_1$, where P_3^A denotes the projective A-module corresponding to the vertex 3 and S_1 the simple A-module corresponding to the vertex 1.

Since $\operatorname{pd}_A S_1 = \infty$ we conclude that $\operatorname{pd}_A L = \infty$.

2. Main results

Let R be a split-by-nilpotent extension of A by the nilpotent ideal E. In this section, we study the relationship between the Gorenstein homological properties of A and the Gorenstein homological properties of R. We start studying the semi-Gorenstein-projective modules over these algebras.

Proposition 2.1. *Let R be a split-by-nilpotent extension of A by the nilpotent ideal E.*

(i) *Assume that $\operatorname{pd}_A R < \infty$ and $\operatorname{id}_A R < \infty$. If N is a semi-Gorenstein-projective A-module, then $R \otimes_A N$ is also a semi-Gorenstein-projective R-module.*
(ii) *Assume that $\operatorname{pd}_R A < \infty$ and $\operatorname{id}_R A < \infty$. If L is a semi-Gorenstein-projective R-module, then $A \otimes_R L$ is also a semi-Gorenstein-projective A-module.*

Proof. We only prove (i) since (ii) follows with similar arguments.

Assume that N is a semi-Gorenstein-projective A-module. Since $\operatorname{pd}_A R < \infty$ and $\operatorname{id}_A R < \infty$, by Lemma 1.2 we have that $\operatorname{Ext}^i_A(N, R) = 0$ and $\operatorname{Tor}^A_i(R, N) = 0$. Let

$$P^\bullet : \cdots \to P_n \to P_{n-1} \to \cdots \to P_1 \to P_0 \to 0 \tag{2.1}$$

be a minimal projective resolution of N.

If we apply the functor $R \otimes_A -$ to (2.1), since $\operatorname{Tor}^A_i(R, M) = 0$, we get the following exact sequence:

$$R\otimes_A P^\bullet : \cdots \to R\otimes_A P_n \to R\otimes_A P_{n-1} \to \cdots \to R\otimes_A P_1 \to R\otimes_A P_0 \to R\otimes_A N \to 0.$$

By [4, Lemma 1.3], we have that $R\otimes_A P^\bullet$ is a projective resolution of $R\otimes_A N$. Since $\operatorname{Hom}_R(R\otimes_A P_i, R) \simeq \operatorname{Hom}_A(P_i, R)$, we have that $\operatorname{Hom}_R(R\otimes_A P^\bullet, R)$ is isomorphic to the sequence

$$0 \to \operatorname{Hom}_A(P_0, R) \to \operatorname{Hom}_A(P_1, R) \to \cdots \to \operatorname{Hom}_A(P_n, R) \to \dots,$$

which is an exact sequence because $\operatorname{Ext}^i_A(M, R) = 0$. Therefore, $R \otimes_A N$ is a semi-Gorenstein-projective R-module. □

Corollary 2.2. *Let R be a split-by-nilpotent extension of A by the nilpotent ideal E.*

(i) *Assume that $\operatorname{pd}_A R < \infty$, $\operatorname{id}_A R < \infty$, $\operatorname{pd}_A DR < \infty$ and $\operatorname{id}_A DR < \infty$.*
   (a) *If N is a Gorenstein-projective A-module, then $R\otimes_A N$ is a Gorenstein-projective R-module.*
   (b) *If N is a Gorenstein-injective A-module, then $\operatorname{Hom}_A(M, R)$ is a Gorenstein-injective R-module.*
(ii) *Assume that $\operatorname{pd}_R A < \infty$, $\operatorname{id}_R A < \infty$, $\operatorname{pd}_R DA < \infty$ and $\operatorname{id}_R DA < \infty$.*
   (a) *If L is a Gorenstein-projective R-module, then $A\otimes_R L$ is a Gorenstein-projective R-module.*
   (b) *If L is a Gorenstein-injective R-module, then $\operatorname{Hom}_R(L, A)$ is a Gorenstein-injective R-module.*

Proof. (i) (a) Since N is a Gorenstein-projective A-module, we have that N and $\operatorname{Tr} N$ are semi-Gorenstein-projective A-modules. By Proposition 2.1, we get that $R \otimes_A N$ and $\operatorname{Tr} N \otimes_A R$ are semi-Gorenstein-projective R-modules. It follows from [4, Lemma 2.1] that $\operatorname{Tr}(R \otimes_A N) \cong \operatorname{Tr} N \otimes_A R$. Therefore, $R \otimes_A N$ is a Gorenstein-projective R-module.

(i) (b) Let N be a Gorenstein-injective A-module. By [5, X, Lemma 1.5], the module $\tau^{-1}N$ is a Gorenstein-projective A-module. The result follows from (i) (a) and the fact that $\tau_R(R \otimes \tau^{-1}N) \cong \operatorname{Hom}_A(R, N)$ (see [4, Lemma 2.1]).

(ii) (a) Since L is a Gorenstein-projective R-module, we have that N and $\operatorname{Tr} N$ are semi-Gorenstein-projective R-modules. Then by Proposition 2.1, we have that $A \otimes_R L$ and $\operatorname{Tr} L \otimes_R A$ are semi-Gorenstein-projective A-modules. Since $\operatorname{id}_R A < \infty$, we have that $\operatorname{Tor}_1^R(A, L) = 0$. By [17, Proposition 3.7], $\operatorname{Tr}(A \otimes_R L) \cong \operatorname{Tr} L \otimes_R A$. Therefore, $A \otimes_A L$ is a Gorenstein-projective A-module.

(ii) (b) Let L be a Gorenstein-injective R-module. By [5, X, Lemma 1.5], $\tau^{-1}L$ is a Gorenstein-projective R-module. Since $\operatorname{id}_R A < \infty$, we have that $\operatorname{Tor}_1^R(A, L) = 0$. It follows from [17, Proposition 3.7] that $\tau_A(A \otimes \tau^{-1}L) \cong \operatorname{Hom}_R(A, L)$. The result is a consequence of (ii) (a). □

Theorem 2.3. *Let R be a split-by-nilpotent extension of A by the nilpotent ideal E. Assume that $\operatorname{pd}_R E < \infty$, $\operatorname{pd}_A E < \infty$, $\operatorname{id}_R DE < \infty$ and $\operatorname{id}_A DE < \infty$. Then A is Gorenstein if and only if R is Gorenstein.*

Proof. Assume that A is Gorenstein. First, we show that $\operatorname{pd}_R DR < \infty$. It follows from Proposition 1.5 that

$$\operatorname{pd}_R DR \leq \operatorname{pd}_A DR + \operatorname{pd}_R A.$$

Since $DR \cong DA \oplus DE$ as A-modules, we have $\operatorname{pd}_A DR = \max\{\operatorname{pd}_A DE, \operatorname{pd}_A DA\}$. By definition of Gorenstein algebras we infer that $\operatorname{pd}_A DA < \infty$. Analogously, since $\operatorname{id}_A DE < \infty$, by Lemma 1.3 we get that $\operatorname{pd}_A DE < \infty$. Then, $\operatorname{pd}_A DR < \infty$.

On the other hand, since $\operatorname{pd}_R E < \infty$ we have $\operatorname{pd}_R A < \infty$, because $\Omega^1(A) \cong E$ in $\operatorname{mod} R$, where $\Omega^1(A)$ denotes the first syzygy. Hence, $\operatorname{pd}_R DR < \infty$.

Now we show that $\operatorname{id}_R R < \infty$. It follows from Proposition 1.5 that

$$\operatorname{id}_R R \leq \operatorname{id}_A R + \operatorname{id}_R DA.$$

Since $R \cong A \oplus E$ as A-modules, we have $\operatorname{id}_A R = \max\{\operatorname{id}_A A, \operatorname{id}_A E\}$. By Lemma 1.3 we get $\operatorname{id}_A E < \infty$, because $\operatorname{pd}_A E < \infty$ and A is Gorenstein. Then, $\operatorname{id}_A R < \infty$.

On the other hand, since the injective envelope of DA in $\operatorname{mod} R$ is DR we get that $\Omega^{-1}(DA) \cong DE$. Therefore, since $\operatorname{id}_R DE < \infty$ we obtain $\operatorname{id}_R DA < \infty$. Hence, $\operatorname{id}_R R < \infty$, concluding that R is Gorenstein.

Conversely, suppose that R is a Gorenstein algebra. By Proposition 1.5, we get that

$$\operatorname{id}_A A \leq \operatorname{id}_R A + \operatorname{id}_A DR.$$

As A-modules, we have that $DR = DA \oplus DE$, which implies that $\operatorname{id}_A DR = \operatorname{id}_A DE < \infty$. It is clear that $\operatorname{pd}_R A = \operatorname{pd}_R M + 1 < \infty$. Since we are assuming that R is Gorenstein, we know that $\operatorname{pd}_R A < \infty$ if and only if $\operatorname{id}_R A < \infty$. Therefore, $\operatorname{id}_A A < \infty$.

It remains to prove that $\operatorname{pd}_A DA < \infty$. By Proposition 1.5, we get that

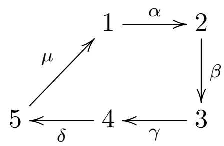
$$\operatorname{pd}_A DA \leq \operatorname{pd}_R DA + \operatorname{pd}_A E.$$

By hypothesis, we know that $\operatorname{pd}_A E < \infty$. Furthermore, we have $\operatorname{id}_R DA = \operatorname{id}_R DE + 1 < \infty$. Since R is Gorenstein, it follows that $\operatorname{pd}_R DA < \infty$ if and only if $\operatorname{id}_R DA < \infty$. Hence, $\operatorname{pd}_A DA < \infty$ and in consequence A is Gorenstein. $\square$

The following example shows that the conditions $\operatorname{id}_R DE < \infty$ and $\operatorname{pd}_R E < \infty$ can not be removed.

Example 2.4. Consider the following algebras:

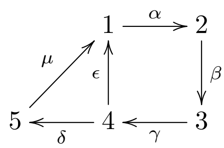
Q_A :



$I_A = \langle \beta\alpha\mu, \gamma\beta\alpha, \mu\delta\gamma\beta, \alpha\mu\delta \rangle, \quad A = kQ_A/I_A$

and

Q_R :



$I_R = \langle \beta\alpha\mu, \gamma\beta\alpha, \mu\delta\gamma\beta, \alpha\mu\delta, \alpha\epsilon, \epsilon\gamma \rangle, \quad R = kQ_R/I_R.$

Observe that R is a split-by-nilpotent extension of A by $E = \langle \epsilon \rangle$. It is not hard to see that A is a Gorenstein algebra and R is not so. As R-modules, $E \cong S_1$ and $DE \cong S_4$. Moreover, $\operatorname{pd}_R E = \infty = \operatorname{id}_R DE$.

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